PATTERNS OF STATIONARY REFLECTION

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Singular cardinals yield surprising results in set theory. After Cohen proved that CH is independent of ZFC, Easton proved that on regular cardinals, the continuum function $\kappa \mapsto 2^\kappa$ is constrained only by the facts that $\lambda \leq \kappa \Rightarrow 2^\lambda \leq 2^\kappa$ and that $\mathrm{cf}(2^\kappa) > \kappa$. In other words, the ZFC constraints on $\kappa \mapsto 2^\kappa$ are fully characterized relative to the class of regular cardinals. In an unexpected turn, Silver proved that GCH cannot fail for the first time at a singular cardinal of uncountable cofinality. In other words, the failure of CH_κ is compact for such cardinals.

We are not limited to studying cardinal arithmetic when considering these compactness phenomena. We can also investigate the compactness of $\square_\kappa$, a canonical property of Gödel's Constructible Universe L. Cummings, Foreman, and Magidor showed that it is consistent for $\square_{\aleph_n}$ to hold for all $n < \omega$ while $\square_{\aleph_\omega}$ fails, and Krueger later showed that it is also consistent for $\square^*_{\aleph_\omega}$ to fail in this situation. However, the question for singulars of uncountable cofinality is still wide open.

For this talk, we will present an Easton-style result for stationary reflection—which is relevant because failure of stationary reflection at κ^+ is an important consequence of $\square_\kappa$. If S is a stationary subset of a cardinal κ, the reflection principle $\mathsf{SR}(S)$ asserts that every stationary subset of S reflects. Assuming the consistency of a supercompact cardinal, we prove that given a fixed $n < \omega$, there are only a few trivial ZFC constraints on $\mathsf{SR}(\kappa \cap \mathrm{cof}(\aleph_n))$ (current work in inner model theory suggests that the large cardinal assumption is close to optimal). The successors of singular cardinals present the greatest hurdle for this result, and require a nonstandard approach to PCF theory.

The Easton-style result on stationary reflection is joint work with Sy-David Friedman. If time permits, we will discuss tentative steps towards addressing the analogous question for Jensen's $\square_\kappa$, which is joint work with both Sy-David Friedman and Dima Sinapova.

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